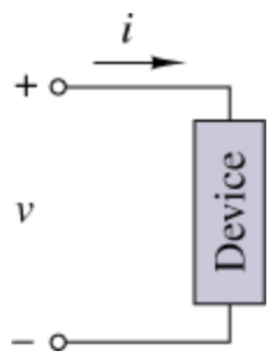
Linear Resistive Circuits

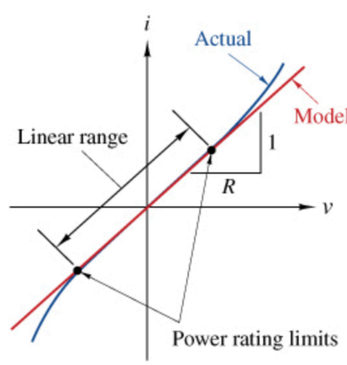
2 Basic Circuit Analysis

2.1 Element Constraints

- A circuit is a collection of interconnected electrical devices. An electrical device is a component treated as a separate entity.
- Two terminal device is described by it's $i - v$ characteristic.
- TO distinguish between an actual device and it's model, we model the circuit **element**.



Linear Resistor



- Equation describing linear resistor is Ohm's Law — $v = iR$
- **Conductance** G is a parameter with unit **siemens** S

$$G = \frac{1}{R}$$

- Linear means the characteristic is a straight line through the origin
- Power associated with the resistor is

$$p = i^2 R = \frac{v^2}{R}$$

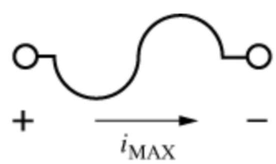
- By the **passive sign convention**, the linear resistor always absorbs or consumes power.

Open and Short Circuits

- Open circuit — $i = 0$ and $R = \infty$
- Short circuit — $v = 0$ and $R = 0$

Fuses

- A fuse is a safety device designed to protect from high current
- Consists of a metal strip mounted between a pair of electrical terminals
    - Housed in noncombustible housing like glass
- If current goes too high, temperature increase will melt the conducting material thus opening the circuit



The Ideal Switch

- Off — $i = 0$, v can take any value
- On — $v = 0$, i can take any value

Ideal Sources

- Two main elements, **voltage sources** and **current sources**
- Ideal voltage source — $v = v_S$ and i can take any value
- Ideal current source — $i - i_S$ and v can take any value
- Voltage or current produced by an ideal source is called a **forcing or driving function** because it represents an input that causes a circuit response

2.2 Connection Constraints

- Laws governing circuit behaviour (**Kirchoff's Laws**) are based on work of German scientist Gustav Kirchoff
    - These are called connection constraints because they are based only on circuit connections, not devices in the circuit
- Circuit — interconnection of electrical devices

- Node — electrical juncture of two or more devices
- Loop — Closed path formed by tracing through an ordered sequence of nodes without passing through a node more than once.

Kirchoff's Current Law

💡 The algebraic sum of the currents entering a node is 0 at every instant.

- We can alternatively express this as "the sum of the currents entering a node equals the sum of the currents leaving the node."
- In a circuit containing N nodes, there are only $N - 1$ independent KCL connection equations.

Kirchoff's Voltage Law

💡 The algebraic sum of all the voltages around a loop is 0 at every instant.

- In a circuit containing E two-terminal elements and N nodes, there are only $E - N + 1$ independent KVL connection equations.

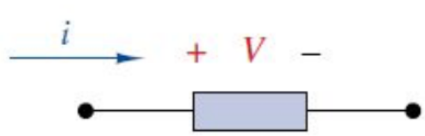
Parallel and Series Connections

- Two elements are in parallel if one can form a loop containing no other elements.
    - Alternatively, they share the same two nodes at both terminals.
- Two elements are in series when they have one common node.

2.3 Combined Constraints

Assigning Reference Marks

- If you pick the plus (+), then the current arrow (—>) must point to the plus.
- If you pick the current arrow (—>), then it must point to the plus (+).



2.4 Equivalent Circuits

- Two circuits are called **equivalent** if they have identical $i - v$ characteristics at a specified pair of terminals.

Current Division

$$i_2 = \left(\frac{R_1}{R_1 + R_2}\right) i_s$$

---

3 Circuit Analysis Techniques

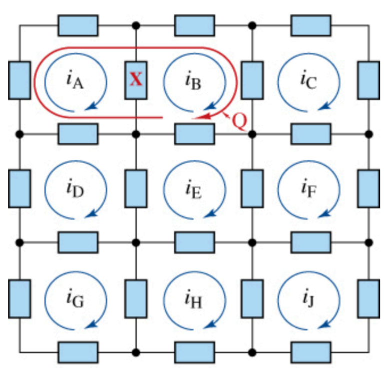
3.1 Node-Voltage Analysis

1. Select a reference node (ground). Identify a node voltage at each of the remaining $N - 1$ nodes and a current with every element in the circuit.
2. Write KCL connection constraints in terms of element currents at $N - 1$ non-reference nodes.
3. Use the $i - v$ relationships of the elements and the fundamental property of node analysis to express the element currents in terms of the node voltages.
4. Substitute the element constraints from step 3 into the KCL connection constraints from step 2 and arrange the resulting $N - 1$ equations in a standard form.

- When voltage sources are involved, we can use supernodes to simplify analysis.

3.2 Mesh-Current Analysis

- A planar circuit can be drawn on a flat surface without crossovers in the window pane.
- To define a set of variables, we associate a mesh current with each of the window panes and assign a reference direction.



1. Identify a mesh current with every mesh and a voltage across every circuit element.
2. Write KVL connection constraints in terms of the element voltages around every mesh.
3. Use KCL and $i - v$ relationships of the elements to express the element voltages in terms of the mesh currents.
4. Substitute the element constraints from step 3 into the connection constraints from step 2 and arrange the resulting equations in standard form.

- If current sources are involved, we can use super-meshes to simplify our analysis.

3.3 Linearity Properties

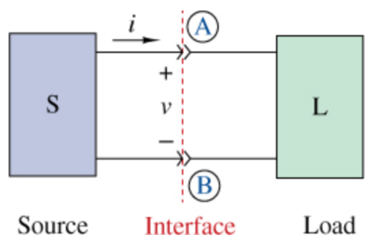
$$f(Kx) = Kf(x) \quad \text{(homogeneity)}$$
$$f(x_1 + x_2) = f(x_1) + f(x_2) \quad \text{(additivity)}$$

Superposition Principle

1. Set all independent sources except one to zero and find the output of the circuit due to that source alone.
2. Repeat step 1 for all the independent sources.
3. The total output is the sum of the contribution of each source acting independently.

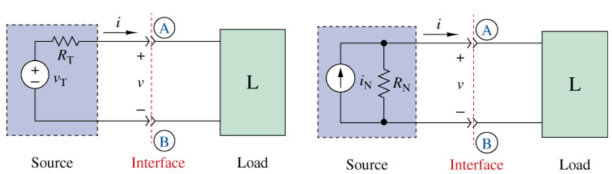
3.4 Thévenin and Norton Equivalent Circuits

- An interface is the connection between two circuits
  - We think of one as the source (S) and the other as the load (L)



- Conditions under which Thévenin and Norton equivalent circuits exist can be stated as a theorem

> 💡 If the source circuit in a two-terminal interface is linear, then the interface signals v and i do not change when the source circuit is replaced by it's Thévenin or Norton equivalent circuit.



Application to Nonlinear Loads

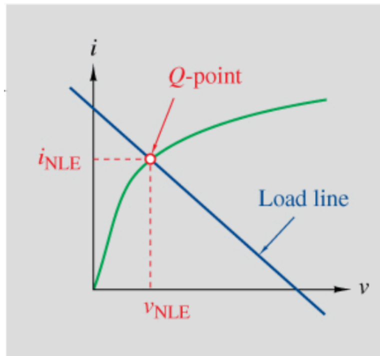
- The $i - v$ relationship of the Thevenin equivalent can be written with interface current as the dependent variable

$$i = \left(-\frac{1}{R_T}\right)v + \left(\frac{v_T}{R_T}\right)$$

- This is the equation of a straight line which intersects the y-axis at i_{SC} and the x-axis at v_{OC}

- While this should be called the source line because it is determined by the Thevenin equivalent of the source circuit, electrical engineers call it the **load line**.

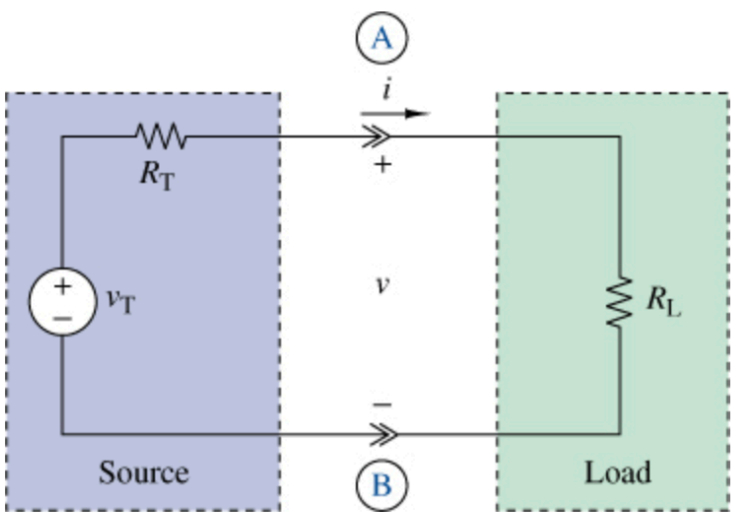
- If we graph this load line with the $i - v$ characteristic of the nonlinear device, we can solve for the intersection point called the **operating point, quiescent point, or Q-point**.
- We usually can't solve these Q-points algebraically, instead we use approximation or nowadays, computational tools.



3.5 Maximum Signal Transfer

- We define the maximum voltage, current, and power available at an interface between a fixed source and an adjustable load.

For the circuit below:



the interface voltage is

$$v = \frac{R_L}{R_L + R_T} v_T$$

The voltage is a max if R_L is made very large, ideally infinite. In that case $v_{MAX} = v_T = V_{OC}$.

The current delivered at the interface is:

$$i = \frac{v_T}{R_L + R_T}$$

It then follows that the maximum possible current is then the short-circuit current i_{SC}.

Then,

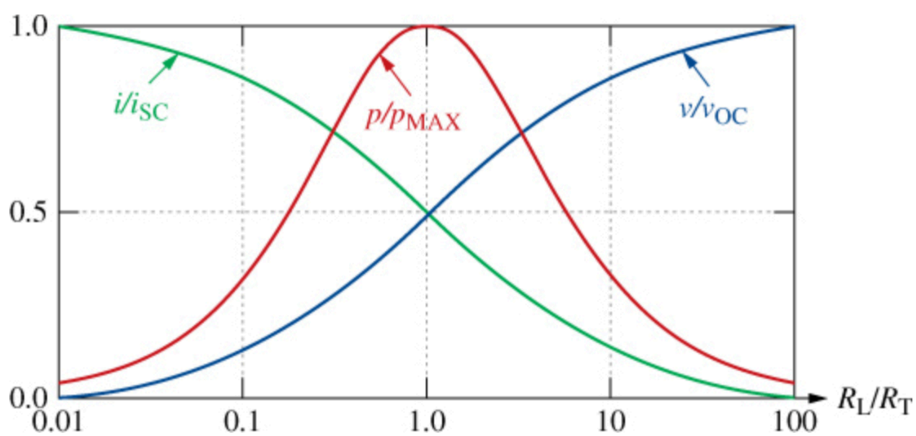
$$p = v \times i = \frac{R_L v_T^2}{(R_L + R_T)^2}$$

For a given source, the conditions for max voltage and current both give zero power (as either voltage or current go to zero). The value of R_L that maximizes the power can be found by differentiating the above equation and solve for R_L where $dp/dR_L = 0$.

$$\begin{aligned} \frac{dp}{dR_L} &= \frac{[(R_L + R_T)^2 - 2R_L(R_L + R_T)]v_T^2}{(R_L + R_T)^4} \\ &= \frac{R_T - R_L}{(R_L + R_T)^3} v_T^2 \\ &= 0 \end{aligned}$$

By solving further, we find that the condition for maximum power transfer is when the load resistance equals the Thevenin resistance, or $R_T = R_L$ (this called **matching**). Then,

$$p_{MAX} = \frac{v_T^2}{4R_T} = \frac{i_N^2 R_T}{4} = \left[\frac{v_{OC}}{2}\right]\left[\frac{i_{SC}}{2}\right]$$

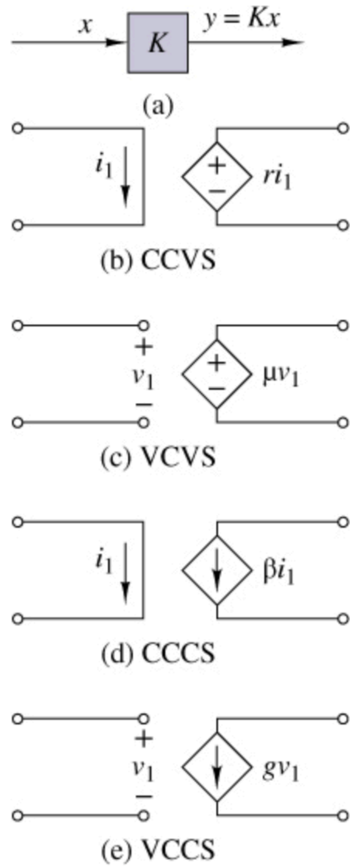


4 Active Circuits

4.1 Linear Dependent Sources

- An **active device** requires an external power supply to operate, and an **active circuit** is one that contains one or more active devices.

- Four possible types of dependent sources:



4.2 Analysis of Circuits with Dependent Sources

- No matter what analysis we do, we must not lose track of the signal(s) that drives the dependent source(s).
    - Methods like node and mesh analysis can be adapted to include dependent sources as well.
- Most techniques/content covered in examples so I won't be including here.

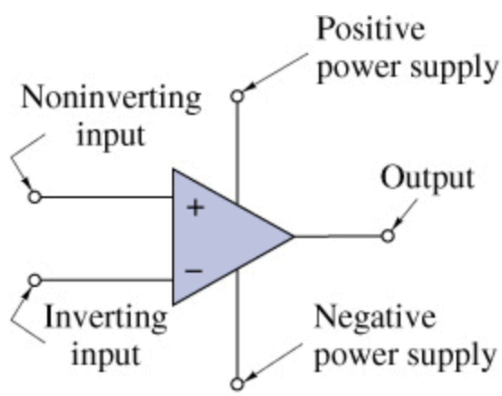
2️⃣

Operational Amplifiers

4 Active Circuits

4.3 The Operational Amplifier

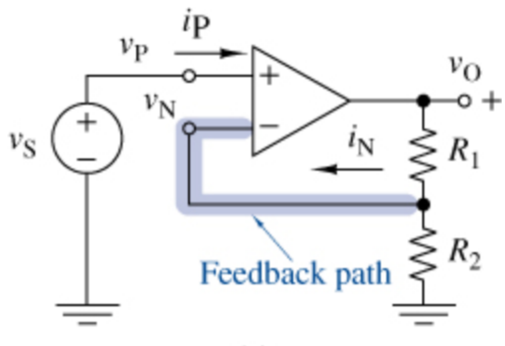
- Op Amps first used in a 1947 National Defence Research Council paper describing high gain amplifier circuits used to carry out mathematical operations.
    - Early op amps were made from vacuum tubes, but by the 70's, the IC version became dominant.



The global KCL equation for the complete set of variables is $i_O = I_{C_+} + I_{C_-} + i_P + i_N$

- Transfer characteristic is divided into three regions or modes: +saturation, -saturation, and linear.
    - +Saturation mode when $A(v_p - v_n) > V_{CC}$ and $v_O = +V_{CC}$
    - -Saturation mode when $A(v_p - v_n) < -V_{CC}$ and $v_O = -V_{CC}$
    - Linear mode when $A|v_p - -v_n| < V_{CC}$ and $v_O = A(-v_p - v_n)$
- The $i - v$ relationships of the ideal model of the op amp are as:
    - $v_P = v_N$
    - $i_P = i_N = 0$

Non-Inverting OP AMP

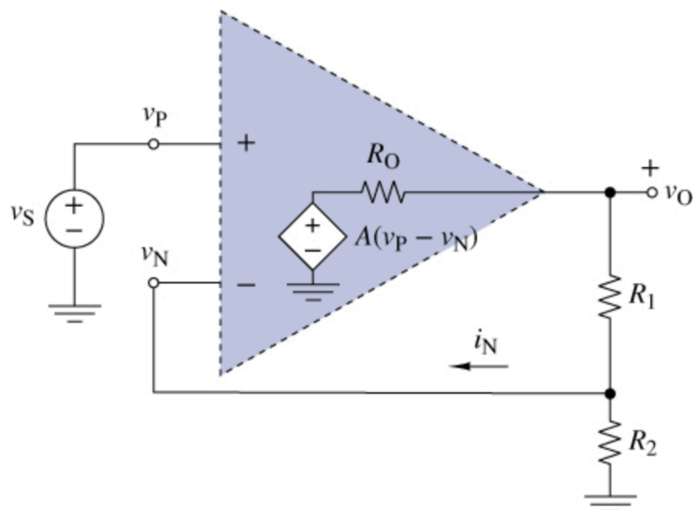


$$v_O = \frac{R_1 + R_2}{R_2} v_S$$

- The proportionality constant K is sometimes called the closed-loop gain because it defines the input-output voltage relationship when the feedback loop is connected (closed).
- There are two types of gains when discussing OP AMP circuits, first is closed-loop gains above.
  - Second is open-loop gain voltage gain provided by the OP AMP device itself.

Effects of Finite OP AMP Gain

- Ideal OP AMP model has infinite gain, but the actual devices have very large but finite gain.



Start by determining the output voltage

$$\begin{aligned} v_O &= \frac{R_1 + R_2}{R_O + R_1 + R_2} A(v_P - v_N) \\ &= \left[\frac{R_1 + R_2}{R_0 + R_1 + R_2}\right] A \left[v_S - \frac{R_2}{R_1 + R_2} v_O\right] \\ &= \frac{A(R_1 + R_2)}{R_O + R_1 + R_2(1 + A)} v_S \end{aligned}$$

Now we take $A \to \infty$

$$v_O = \frac{R_1 + R_2}{R_2} v_S = K v_S$$

4.4 OP AMP Circuit Analysis

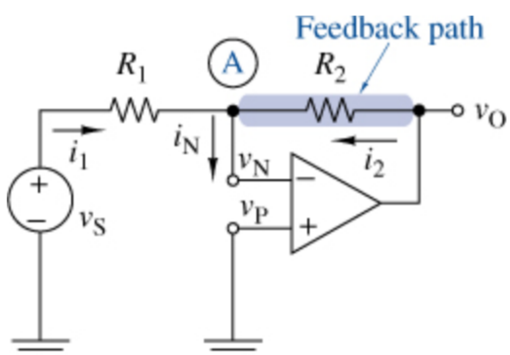
- OP AMP circuit analysis uses advantage of OP AMPS being connected in cascade (like in series)

Voltage Follower

- Also called a buffer
- Feedback is a direct path to the inverting input
- Since there is no input current $i_P = 0$ there is no voltage across R_S and thus $v_P = v_N$
    - Hence $v_O = v_S$ and the name is the voltage "follower"
- This is used in interface circuits to separate the source from the load.

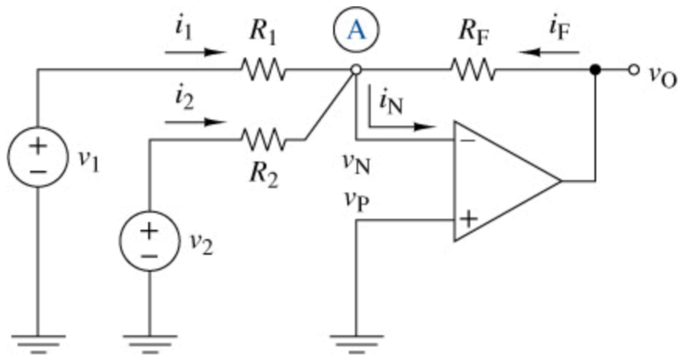
The Inverting Amplifier

$$v_O = -\left(\frac{R_2}{R_1}\right) v_S$$



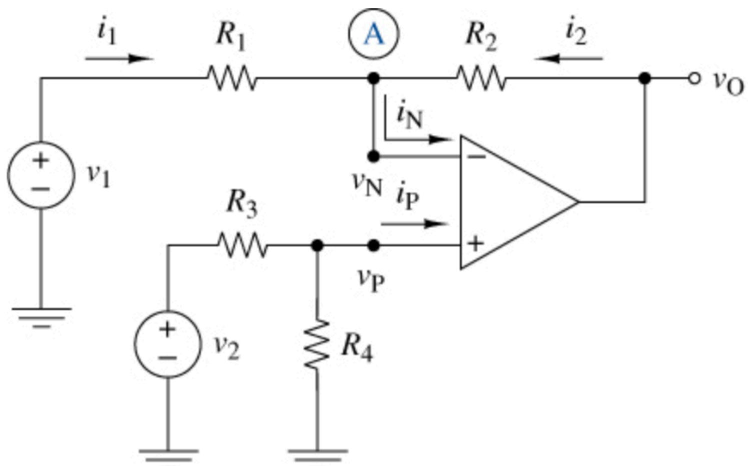
The Summing Amplifier

$$v_O = \left(-\frac{R_F}{R_1}\right) v_1 + \left(-\frac{R_F}{R_2}\right) v_2$$



The Differential Amplifier

$$v_O \frac{R_2}{R_1}(v_2 - v_1)$$



Node-Voltage Analysis with OP AMP Circuits

1. Identify a node voltage at all non-reference nodes, including OP AMP outputs, but do not formulate node equations at the OP AMP output nodes.
2. Formulate node equations at the remaining non-reference nodes and then use the ideal OP AMP voltage constraint $v_P = v_N$ to reduce the number of unknowns.

3️⃣

Linear Dynamic Circuits

6 Capacitance and Inductance

6.1 The Capacitor

- Capacitors are dynamic elements involving time variation of an electric field produced by a voltage.
- A uniform electric field $\vec{E}(t)$ exists between the metal plates when a voltage exists across the capacitor.
- When the separation d is small compared to the dimension of the plates, the electric field between the plates is

$$\vec{E}(t) = \frac{q(t)}{\varepsilon A}$$

- where ε is the permittivity of the dielectric, A is the area of the plates, and $q(t)$ is the magnitude of the electric charge on each plate.
- The relationship between the electric field and voltage across the capacitor is given by:

$$\vec{E}(t) = \frac{v_C(t)}{d}$$

- The proportionality constant inside the brackets in this equation is the capacitance C, which is by definition:

$$C = \frac{\varepsilon A}{d}$$

$i - v$ Relationship

$$\frac{dq(t)}{dt} = \frac{d[Cv_C(t)]}{dt}$$

In practice, the time t_0 is established by a physical event such as closing a switch or the start of a particular clock pulse.

$$v_C(t) = v_C(0) + \frac{1}{C}\int_0^t i_C(x)\, dx$$

By the passive sign convention, the power in dissipated by a capacitor is given by:

$$p_C(t) = i_C(t)\, v_C(t)$$

To determine the stored energy, we take the first derivative. Since power is the time rate of change of energy, the quantity inside the brackets must be the energy stored in the capacitor.

$$w_C(t) = \frac{1}{2}Cv_C^2(t)$$

- Stored energy is never negative and is proportional to the square of the voltage.
- The relationship also implies voltage is a continuous function of time.
- Since power is the time derivative of energy, a discontinuous change in energy implies infinite power, which is physically impossible.
- The capacitor voltage is a **state variable** because it determines the energy state of the element.

To summarize, the capacitor is a dynamic circuit element with the following properties:

1. The current through the capacitor is zero unless the voltage is changing. The capacitor acts like an open circuit to DC excitations.
2. The voltage across the capacitor is a **continuous function** of time. A discontinuous change in capacitor voltage would require infinite current and power, which is physically impossible.
3. The capacitor absorbs power from the circuit when storing energy and returns previously stored energy when delivering power. The net energy transfer is nonnegative, indicating that the capacitor is a passive element.

6.2 The Inductor

- The inductor is a dynamic circuit element involving the time variation of a magnetic field produced by a current.
- In a linear magnetic medium, the flux is proportional to both the current and the number of turns in the coil.
- The total flux is given by:

$$\phi(t) = k_1 N i_L(t)$$

- k_1 is a constant of proportionality dependent on the coil material, it's geometry, and its permeability, μ
- Flux linkage in a coil is given by the symbol λ with units weber-turns. It is proportional to the number of turns in the coil and to the total magnetic flux

$$\lambda(t) = N\phi(t)$$

- The proportionality constant inside the brackets in the inductance L, given by:

$$L = k_1 N^2$$

- The unit of inductance is the henry (H)

$$\lambda(t) = Li_L(t)$$

$i - v$ Relationship

$$\frac{d[\lambda(t)]}{dt} = \frac{d[Li_L(t)]}{dt}$$

$$v_L(t) = L\frac{di_L(t)}{dt}$$

The reference time t_0 is established by some physical event such as closing or opening a switch. Without losing any generality, we assume $t_0 = 0$ and solving for $i_L(t)$ in the form

$$i_L(t) = i_L(0) + \frac{1}{L}\int_0^t v_L(x)\, dx$$

Power and Energy

Just as for the capacitor, we have power given by:

$$p_L(t) = i_L(t)\, v_L(t)$$

As is with capacitor energy, the constant in this expression is zero since it is energy stored in an instant t. Thus we have:

$$w_L(t) = \frac{1}{2}Li_L^2(t)$$

In this case, **current** is the state variable of the inductor as it determined the energy state of the element.

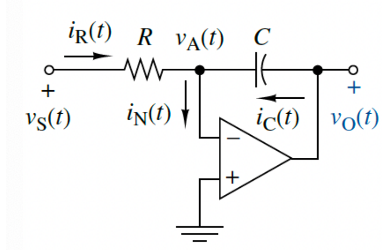
To summarize, we have the following points for the inductor:

1. The voltage across the inductor is zero unless the current through it is changing. For DC excitations, the inductor is a short circuit.
2. The current through the inductor is a continuous function of time. A discontinuous change would require infinite voltage and power, which is physically impossible.
3. The inductor absorbs power from the circuit when storing energy and delivers power to the circuit. The net energy is nonnegative, indicating that the inductor is a passive element.

More About Duality

KVL	↔	KCL
Loop	↔	Node
Resistance	↔	Conductance
Voltage source	↔	Current source
Thévenin	↔	Norton
Short circuit	↔	Open circuit
Series	↔	Parallel
Capacitance	↔	Inductance
Flux linkage	↔	Charge

6.3 Dynamic OP AMP Circuits



Let's determine the signal processing function of the circuit. Start by writing the KCL equation at node A.

$$i_R(t) + i_C(t) = i_N(t)$$

Write the resistor and capacitor device equations using the fundamental property of node voltages.

$$\begin{cases} i_C(t) = C\frac{d[v_O(t) - v_A(t)]}{dt} \\ i_R(t) = \frac{1}{R}[v_S(t) - v_A(t)] \end{cases}$$

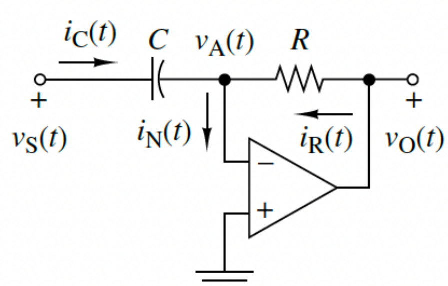
Then we substitute the element constraints into the KCL connection constraint produces

$$\frac{v_S(t)}{R} + C\frac{dv_O(t)}{dt} = 0$$

By integrating and rearranging a bit, we can find that:

$$v_O(t) = v_O(0) - \frac{1}{RC}\int_0^t v_S(x)\,dx$$

Similarly, we have an inverting differentiator circuit



6.4 Equivalent Capacitance and Inductance

Capacitors

$$C_{EQ} = C_1 + C_2 + \cdots + C_N \text{ (parallel connection)}$$

$$\frac{1}{C_{EQ}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N} \text{ (series connection)}$$

Inductors

$$\frac{1}{L_{EQ}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N} \text{ (parallel connection)}$$

$$L_{EQ} = L_1 + L_2 + \cdots + L_N \text{ (series connection)}$$

DC Equivalent Circuits

- Under DC conditions, a capacitor acts as a open circuit and an inductor acts as a short circuit.
- DC capacitor voltage and inductor current becomes initial conditions for a transient response that begins at $t = 0$ when something in the circuit changes.

---

7 First- and Second-Order Circuits

7.1 RC and RL Circuits

- RC and RL circuits are called **first-order circuits** because their behaviour is governed by a first-order DE.

Zero-Input Response of First-Order Circuits

The response depends on three factors:

1. The inputs driving the circuit $v_T(t)$
2. The values of the circuit parameters R_T and C
3. The value of $v(t)$ at $t = 0$ (initial condition)

The classical approach is to try an exponential solution

$$v(t) = Ke^{st}$$

7.2 First-Order Circuit Step Response

- Designing a circuit to meet transient response specifications requires making compromises with respect to the circuit's steady-state performance.
- Because the circuit is linear, we choose a method that uses superposition to divide the solution for $v(t)$ into two components:
  - $v_N(t)$ is the natural response and is the general solution when the input is set to zero
  - $v_F(t)$ is the forced responde, which is a particular solution when the input is a step function

$$v(t) = v_N(t) + v_F(t)$$

The step response of a first-order circuit depends on three quantities:

1. The amplitude of the step input (V_A or I_A)
2. The circuit time constant (R_TC or L/R_N)

3. The value of the state variable at $t = 0$ (V_0 or I_0)

Zero-State Response

- The zero-state response is proportional to the amplitude of the input step function.
- The total response is not directly proportional to the input amplitude.
- The zero-state response occurs when the input is zero ($V_A = 0$ or $I_A = 0$)

7.3 Initial and Final Conditions

$$x(t) = [x(0) - x(\infty)]e^{-t/T_C} + x(\infty)$$

The state variable response in switched dynamic circuits is found using the following steps:

1. Find the initial value by applying DC analysis to the circuit configuration for $t < 0$with the element replaced with an open circuit.
2. Find the final value by applying DC analysis to the circuit configuration for $t > 0$ with the element replaced with an open circuit.
3. Find the time constant T_C of the circuit in the configuration for $t > 0$.
4. Write the step response directly without formulating and solving for the circuit differential equation.

7.5 The Series RLC Circuit

- Second order circuits contain two energy storage elements that cannot be replaced by a single equivalent element.
  - They are called second-order circuits
- There are two types, series and parallel RLC circuits

Solve for the roots of the characteristic equation

$$LCs^2 + R_TCs + 1 = 0$$

- The solutions then define the natural frequencies of the circuit
- We can have:
  - s as the complex frequency
  - α as the neper frequency
  - β as the radian frequency or the damped natural frequency

7.6 The Parallel RLC Circuit

Characteristic equation

$$LCs^2 + \frac{L}{R_N}s + 1 = 0$$

7.7 Second-Order Circuit Step Response

General second-order differential equation with a step function has the form

$$a_2\frac{d^2y(t)}{dt^2} + a_1\frac{dy(t)}{dt} + a_0y(t) = Au(t)$$

The step response is found by the partitioning $y(t)$ into forced and natural components:

$$y(t) = y_N(t) + y_F(t)$$

In a second order circuit, the zero-state and natural responses studied take one of the three possible forms:

- overdamped,
- critically damped,
- underdamped

Now we need to introduce two new parameters

$$\omega_0^2 = \frac{a_0}{a_2} \text{ and } 2\zeta\omega_0 = \frac{a_1}{a_2}$$

- ω_0 is the undamped natural frequency
- ζ is the damping ratio

Now we can write the general homogeneous equation in the form:

$$\frac{d^2y_N(t)}{dt^2} + 2\zeta\omega_0\frac{dy_N(t)}{dt} + \omega_0^2y_N(t) = 0$$

From this we can get

$$\omega_0 = \sqrt{\frac{1}{LC}}$$

and

$$\zeta = \frac{R_T}{2L\omega_0}$$

7.8 Summary

- Circuits containing one storage element are described by first-order differential equations
- The zero-input response in a first-order circuit is an exponential whose time constant depends on circuit parameters.
- The total response is the sum of the forced and natural responses.

- The zero-state response results from the input driving forces, and the zero-input response is caused by the initial energy stored in the storage element.
- For a sinusoidal input, the forced response is called the sinusoidal steady-state response, or the AC response.
- Circuits containing two energy storage elements are called second-order circuits
- Circuit damping ratio ζ and undamped natural frequency ω_0 determine the form of the zero-input and natural responses of any second-order circuit.

4️⃣

Sinusoidal Steady State Analysis

8 Sinusoidal Steady-State Response

8.1 Sinusoids and Phasors

- Phasor is a complex number representation for the amplitude and phase angle of sinusoidal voltage or current

$$\mathbb{V} = V_A e^{j\phi} = V_A \cos\phi + jV_A \sin\phi$$

- Absence of frequency information in the phasors results from the fact that in the sinusoidal steady state, all currents and voltages are sinusoids with the same frequency

Properties of Phasors

$$\mathbb{V} = \mathbb{V}_1 + \mathbb{V}_2 + \dots \mathbb{V}_n$$

8.2 Phasor Circuit Analysis

- Kirchhoff's current and voltage laws still apply to phasors
- Phasors with the same phase angle are said to be in phase, otherwise, they considered out of phase.

The Impedance Concept

- Impedance is the proportionality constant relating phasor voltage and current
  - Resistor: $Z_R = R$
  - Capacitor: $Z_C = \frac{1}{j\omega C}$
  - Inductor: $Z_L = j\omega L$
- Impedance is a complex number, it's not a phasor

8.3 Basic Phasor Circuit Analysis and Design

1. The circuit is transformed into the phasor domain by representing the input and response sinusoids as phasors and the passive circuit elements by their impedance
2. Standard algebraic circuit analysis techniques are applied to solve the phasor domain circuit for the desired unknown phasor responses
3. The phasor responses are inverse transformed back into time domain sinusoids to obtain the response waveforms

Series Equivalence and Voltage Division

The equivalent impedance Z_{EQ} is a complex quantity of the form

$$Z_{EQ} = R + jX$$

where R is the real part and X is the imaginary part. R is resistance and X is reactance.

A positive X is called an inductive reactance, while a negative X is called a capacitive reactance.

Parallel Equivalence and Current Division

$$Y = \frac{1}{Z} = G + jB$$

Inverse impedance is called **admittance** Y, with real part being called conductance G and imaginary part being called susceptance B

Summary

- A phasor is a complex number representing a sinusoidal waveform. Phasors do not provide frequency information.
- The additive property states that adding phasors is equivalent to adding sinusoids of the same frequency.
- Impedance can be defined as ratio of phasor voltage over phasor current, has same units as resistance
- Frequency at which an equivalent impedance is purely real is called a resonant frequency
- Instantaneous power to a passive element is a periodic function at twice the frequency of the driving force

15 Mutual Inductance and Transformers

15.1 Coupled Inductors

- A DC current produces a constant magnetic field or flux that emanates from the wire
- Faraday's law states that voltage across the inductor equals the time rate of change of the total flux linkage
- If a second inductor is brought close to the first so that the first from the first inductor links with the turns of the second inductor, then the flux linkage will generate a voltage in the second inductor
    - The magnetic coupling between the changing current in one inductor and the voltage generated in a second inductor produces mutual inductance

$$v_1(t) = L_1 i_1'(t) \pm M i_2'(t)$$
$$v_2(t) = \pm M i_1'(t) + L_2 i_2'(t)$$

15.2 The Dot Convention

Mutual inductance is additive when both current reference directions point toward or both point away from dotted terminals; otherwise, it is subtractive.

15.3 Energy Analysis

The energy stored in a pair of coupled inductors is positive if

$$L_1 L_2 \geq M^2$$

The above constraint is written in terms of a new parameter called the coupling coefficient k

$$k = \frac{M}{\sqrt{L_1 L_2}} \leq 1$$

The condition $k = 1$ requires perfect coupling in which all of the flux produced by one inductor links the other.

15.4 The Ideal Transformer

- transformer is an electrical machine that utilizes magnetic coupling between two inductors
- the winding connected to the source is called the primary winding, and the one connected to the load is called the secondary winding
- purpose of the device is to change voltage and current so conditions at the source and load are compatible
- design involves two primary goals:
  - maximize the magnetic coupling between the two windings
  - minimize the power loss in the winding
- the **ideal transformer** is a circuit that is assumed to have perfect coupling and zero power loss

Perfect Coupling

- **perfect coupling** means that all of the flux in the first winding links the second, and vice versa

$$\Phi_1(t) = k_1 N_1 i_1(t)$$

- $\Phi_{12}(t)$ is the flux intercepting winding 1 due to the current in winding 2

$$\frac{v_2(t)}{v_1(t)} = \pm \frac{N_2}{N_1} = \pm n$$

- n is the turns ratio
  - when > 1, the device is called a step-up transformer
  - when < 1, the device is called a step-down transformer

Zero Power Loss

- zero power loss requires that $v_1(t) i_1(t) + v_2(t) i_2(t) = 0$

$i - v$ Characteristics

- when the reference directions for the currents are both toward or both away from the dotted terminals, then $V_2 = +nV_1$ and $I_2 = -I_1/n$; otherwise $V_2 = -nV_1$ and $I_2 = I_1/n$

- on the power side we have

$$Z_{IN} = \frac{\mathbf{V}_1}{\mathbf{I}_1}$$

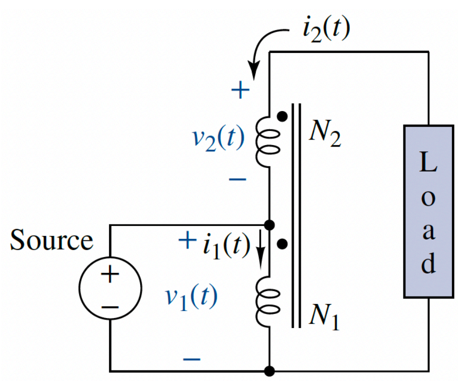
- whereas on the secondary side, we don't follow the passive sign convention, so

$$Z_L = \frac{\mathbf{V}_2}{-\mathbf{I}_2}$$

- from this we can get

$$Z_{IN} = \frac{Z_L}{n^2}$$

- in a transformer, the primary and secondary windings are magnetically coupled but are usually electrically isolated. transformer windings in some applications are configured what's called an autotransformer



16 AC Power Systems

16.1 Average and Reactive Power

$$p(t) = \left[\frac{V_A I_A}{2}\cos\theta\right] + \left[\frac{V_A I_A}{2}\cos\theta\right]\cos 2\omega t + \left[\frac{V_A I_A}{2}\cos\theta\right]\sin 2\omega t$$

- we define the average value of a periodic waveform as

$$P = \frac{1}{T}\int_0^T p(t)\, dt$$

- the average value of $p(t)$ denotes as P is equal to the constant or DC term and has the unit watts W

$$P = \frac{V_A I_A}{2}\cos\theta$$

- the amplitude factor is called the **reactive power** of $p(t)$ where reactive power Q has the units Volt-Amperes Reactive or VAR

$$Q = \frac{V_A I_A}{2} \sin 2\omega t$$

16.2 Complex Power

$$P = V_{rms} I_{rms} \cos\theta$$
$$Q = V_{rms} I_{rms} \sin\theta$$

- we now introduce a new variable called **complex power** (S). at a two-terminal interface it is defined as

$$S = \mathbf{V}\mathbf{I}^*$$

- we could use Euler's identity to write complex power as

$$S = P + jQ$$

- S has units of Volt-Amperes, or VA
- apparent power

$$|S| = V_{rms}\ I_{rms}$$

- power factor

$$\text{pf} \ = \frac{P}{|S|} = \cos\theta$$

- θ is then called the **power factor angle**

$$R = \frac{P}{I_{rms}^2} \text{ and } X = \frac{Q}{I_{rms}^2}$$

16.3 Single-Phase Circuit Analysis

- we can make use of the principle of the conservation of complex power:
  - in a linear circuit the sum of the complex powers produced by all the active sources is equal to the sum of the complex powers delivered to all of the passive loads.
- result is that the average power "produced by" a source or "delivered to" a load has the same sign

16.4 Single-Phase Power Flow

- a **power flow problem**, the complex power to a load is specified, and unknowns are voltages and currents that will make this power flow happen

Power Factor Correction

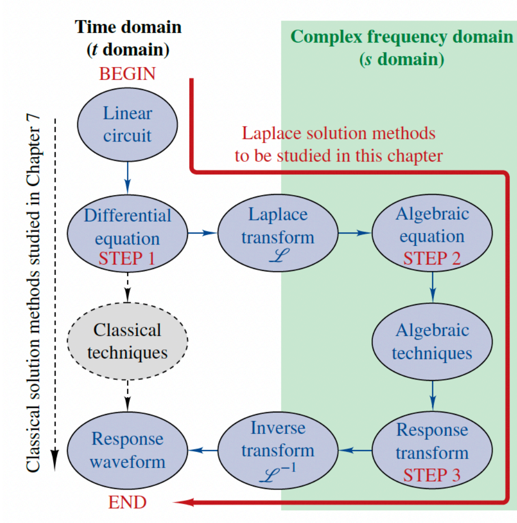
- power factor correction is a process that increases the power factor without changing the power flow to an inductive load
- reactive power Q_C is always negative while Q_L for an inductive load is always positive
    - net decrease in reactive power means that the power factor of the parallel combination is higher than the power factor of the inductive load acting alone
    -

5️⃣

Frequency Domain Analysis

9 Laplace Transforms

9.1 Signal Waveforms and Transforms



- in the time domain, the signal is characterized by its **waveform**, and in the complex frequency domain, the signal is represented by its **transform**

$$F(s) = \int_{0^-}^{\infty} f(t)e^{-st}\, dt$$

- **piecewise continuous** means that the function has a finite number of steplike discontinuities in any finite interval
- **exponential order** means thats the constants K b exist such that $|f(t) < Ke^{bt}$ for all $t > 0$

Inverse Transformation

$$f(t) = \frac{1}{2\pi j} \int_{\alpha - j\infty}^{\alpha + j\infty} F(s)\, e^{st}\, ds$$

9.5 Circuit Response Using Laplace Transforms

1. develop the circuit differential equation in the time domain
2. transform this equation into the s domain and algebraically solve for the response transform

3. apply the inverse transformation to this transform to produce the response waveform

10 s-Domain Circuit Analysis

10.1 Transformed Circuits

1. the process begins with a linear circuit in the time domain. we transform the circuit directly into the s domain
2. once the circuit is transformed in the s domain, we can write the circuit equations directly in that domain using algebra. we can then solve these algebraic equations for the response transform
3. the inverse Laplace transformation then producers the response waveform

11 Network Functions

11.1 Definition of a Network Function

$$\text{network function } = \frac{\text{zero-state response transform}}{\text{input signal transform}}$$

- note this definition specifies zero initial conditions and implies one input
- in a stable circuit, those elements in the forced response that do not decay to zero are called the stead-state response

11.2 Network Functions of One- and Two-Port Circuits

- a driving-port impedance relates the voltage and current at a pair of terminals called a port
- the driving-point impedance seen at a pair of terminals determines the loading effects that result when those terminals are connected to another circuit
- a transfer function relates an input and response at different ports in the circuit

(a) $T_V(s) = \text{Voltage Transfer Function} = \frac{V_2(s)}{V_1(s)}$

(b) $T_I(s) = \text{Current Transfer Function} = \frac{I_2(s)}{I_1(s)}$

(c) $T_Y(s) = \text{Transfer Admittance} = \frac{I_2(s)}{V_1(s)}$

(d) $T_Z(s) = \text{Transfer Impedance} = \frac{V_2(s)}{I_1(s)}$

- signal processing circuits often involve a cascade connection in which the output voltage of one circuit serves as the input to the next stage.

11.3 Network Functions and Impulse Response

- the impulse response is the zero-state response of a circuit when the driving force is a unit impulse applied at $t = 0$

- a linear circuit whose impulse response ultimately returns to zero is said to be asymptotically stable, meaning that the impulse response has a finite time duration

11.4 Network Functions and Step Response

- the step response is the zero-state response of the circuit output when the driving force is a unit step function applied at $t = 0$

$$G(s) = \frac{k_0}{s} + \frac{k_1}{s - p_1} + \frac{k_2}{s - p_2} + \cdots + \frac{k_N}{s - p_N}$$

- the first term, $\frac{k_0}{s}$ is called the forced pole, while the others are called the natural poles

11.5 Network Functions and Sinusoidal Steady-State Response

- when a stable, linear circuit is driven by a sinusoidal input, the output contains a steady-state component that is a sinusoid of the same frequency as the input

$$Y(s) = \frac{k}{s - j\omega} + \frac{k^*}{s - j\omega} + \frac{k_1}{s - p_1} + \frac{k_2}{s - p_2} + \cdots + \frac{k_N}{s - p_N}$$

- the first two terms $\frac{k + k^*}{s - j\omega}$ are the forced poles, while the remainders are called the natural poles

12 Frequency Response

12.1 The Electromagnetic Spectrum and Frequency-Response Descriptors

- R, L, C components that we've worked with so far are called lumped components, meaning an element that is smaller than the wavelength of the wavelength of the applied signal
- when lumped components no longer work, one uses distributed elements
- the range of frequencies with nearly constant gain is called a **passband**
- the range of frequencies with greatly reduced gain is called a **stopband**
- the frequency associated with the transition from a passband to an adjacent stopband is called the **cutoff frequency** ω_C

12.3 First-Order Low-Pass and High-Pass Responses

First-Order Low-Pass Response

$$T(s) = \frac{K}{s + \alpha}$$

- α must be positive so that the natural pole is in the left half of the s-plane
- at $\omega = \alpha$ there is a "corner" so we call it the **corner frequency**

- straight-line approximations need two parameters: α for corner frequency and $T(0)$ for passband gain

Gain-Bandwidth Product (GB)

- given by

$$GB = A\omega_c$$

First-Order High-Pass Response

$$T(s) = \frac{Ks}{s+\alpha}$$

12.4 Bandpass and Bandstop Responses

Bandpass Response

- cascade connection of first-order high- and low-pass circuits
- when second stage doesn't load the first, the overall transfer function is given by

$$T(s) = T_1(s) \times T_2(s)$$

- low frequency $(\omega << \alpha_1 << \alpha_2)$

$$|T(j\omega)| = \frac{|K_1||K_2|\omega}{\alpha_1\alpha_2}$$

- high frequency $(\alpha_1 << \alpha_2 << \omega)$

$$|T(j\omega)| = \frac{|K_1||K_2|}{\omega}$$

- mid frequency $(\alpha_1 << \omega << \alpha_2)$

$$|T(j\omega)| = \frac{|K_1||K_2|}{\alpha_2}$$

Bandstop Response

- frequency response given by

$$T(s) = T_1(s) + T_2(s)$$

- low frequency $(\omega << \alpha_1 << \alpha_2)$

$$|T(j\omega)| = \frac{|K_2|}{\alpha_2}$$

- high frequency $(\alpha_1 << \alpha_2 << \omega)$

$$|T(j\omega)| = |K_1|$$

- mid frequency $(\alpha_1 << \omega << \alpha_2)$

$$|T(j\omega)| \approx 0$$

12.5 The Frequency Response of RLC Circuits

Series RLC Bandpass Circuit

$$T(s) = \frac{R}{R + Z_{LC}(s)}$$

- the maximum gain occurs at the centre frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\omega_{C1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$
$$\omega_{C2} = +\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_C = \sqrt{\omega_{C1}\omega_{C2}}$$

- the bandwidth B is fount by:

$$B = \omega_{C2} - \omega_{C1} = \frac{R}{L}$$

- for historical reasons we add a fifth parameter called the **quality factor** Q, defined as the centre frequency over bandwidth

$$Q = \frac{\omega_0}{B}$$

The Series RLC Bandstop Circuit

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{Z_{LC}(s)}{R + Z_{LC}(s)}$$

The Parallel RLC Bandpass Circuit

$$T(s) = \frac{I_2(s)}{I_1(s)} = \frac{1/R}{1/R + Y_{LC}(s)}$$

14 Active Filter Design

14.1 Active Filters

- passive filters contain only RLC elements
- active filters contain only resistors, capacitors, and OP AMPs.
- they have several advantages:
    - they allow for shape of the response to be tailored with steep roll-offs (high Q) and frequency selectivity comparable to passive RLC circuits
    - they have OP AMP outputs, meaning that the chain rule applies in a cascade design and can be designed with passband gains greater than 1
    - they do not require inductors which can be large, lossy, and expensive
- low-pass filters have general form:

$$T(s) = \frac{K}{(s/\omega_0)^2 + 2\zeta(s\omega_0) + 1}$$

- high-pass filters have general form:

$$T(s) = \frac{K(s/\omega_0)^2}{(s/\omega_0)^2 + 2\zeta(s\omega_0) + 1}$$

- bandpass filters have general form:

$$T(s) = \frac{K(s/\omega_0)}{(s/\omega_0)^2 + 2\zeta(s\omega_0) + 1}$$

- bandstop filters have general form:

$$T(s) = \frac{K[(s/\omega_0)^2 + 1]}{(s/\omega_0)^2 + 2\zeta(s\omega_0) + 1}$$