Vibrations & Time Response → $\frac{x_0}{\dot{x}_0} = \frac{\tan\varphi}{\omega_n}$, $\omega_d = \omega_n\sqrt{1-\zeta^2}$, $a_{max} = \omega_n^2\,(x_{disp})$, $x_{max} = \frac{F_0/k}{1-\left(\frac{\omega}{\omega_n}\right)^2}$

Free undamped → $m\ddot{x} + kx = 0$

$\omega_n = \sqrt{\frac{k}{m}}$, $x = C\sin(\omega_n t + \varphi)$

$\zeta = \frac{c}{2m\omega_n}$ $\quad \frac{x_2}{x_1} = e^{-2\pi\zeta}$ $\quad f = \frac{\omega}{2\pi}$

Period: $\tau_d = \frac{2\pi}{\omega_d}$

Free Damped → $m\ddot{x} + c\dot{x} + kx = 0$

PARALLEL → Σk_i

SERIES → $\Sigma \frac{1}{k_i}$

$M = \frac{x}{\delta} = \frac{x}{F_0/k} = \frac{1}{1-\left(\frac{\omega}{\omega_n}\right)^2}$

$\ddot{x} + 2\zeta\omega_n + \omega_n^2 x = 0$ ASQUARE

Critically Damped → $\zeta = 1$ → $x = (A_1 + A_2 t)\,e^{-\omega_n t}$

Underdamp → $\zeta < 1$ → $x = C\sin(\omega_d t + \varphi)\,e^{-\zeta\omega_n t}$

Overdamp → $\zeta > 1$ → $x = e^{-\zeta\omega_n t}\left(Ae^{\omega_d t} + Be^{-\omega_d t}\right)$

Forced → $\ddot{x} + \omega_n^2 x = \frac{F_0}{m}\sin\omega t$

Constant Acceleration Equations →

$v = v_0 + at$, $s = s_0 + v_0 t + \frac{1}{2}at^2$

$v^2 = v_0^2 + 2a\Delta s$, $v\,dv = a\,ds$

$ds = v\,dt$, $a\,dt = dv$

Normal-Tangential →

$a = \dot{v}\hat{e}_t + \frac{v^2}{r}\hat{e}_n$, $a_t = \dot{v} = r\ddot{\theta}$

$a_n = \frac{v^2}{r} = \rho\dot{\theta}^2 = v\dot{\theta}$

Polar Coordinates → $\quad F = \frac{mv^2}{r}$

$v = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$

$a = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$

$a_\theta = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$

Work-Energy → $T_1 + V_1 + U_{1\to 2} = T_2 + V_2$

$T = \frac{1}{2}mv^2$ $\quad$ For rotating body →

$V_e = \frac{1}{2}k\Delta x^2$ $\quad T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

OR

$V_g = mgh$ $\quad T = \frac{1}{2}I_{IC}\omega^2$

$\Sigma T_i = \Sigma\frac{1}{2}m_i v_i^2 = \frac{1}{2}mv_G^2 + \Sigma\frac{1}{2}m_i|\dot{\rho}_i|^2$

$\bar{r}_G = \frac{\Sigma m_i r_i}{\Sigma m_i}$

$U_{1\to 2} = \int F\cdot dr = \int F\cos\theta\,dr$

★ Work by moment → $U = \int M\,d\theta$

Impulse-Momentum → $G_1 + \int_{t_1}^{t_2}\Sigma F\,dt = G_2$

$\Sigma F = \dot{G}$, $\Sigma M_0 = \dot{H}_0$

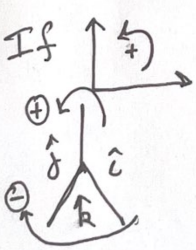
$H_{0_1} + \int_{t_1}^{t_2}\Sigma M_0\,dt = H_{0_2}$

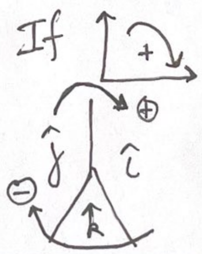
$G = mv$

$\Sigma M_0 = I_G\alpha + ma_G d$, or if $0 = IC$, $\Sigma M_{IC} = I_{IC}\alpha$

$H_0 = I_0\omega = rmv$, or if 0 on body, $H_0 = I_G\omega + m\bar{v}d$

★ Parallel Axis Theorem → $I_A = I_G + md^2$ ★ Watch for mass imbalance

If 

If 

Work of couple moment $= 0$

Work done by friction w/o slip $= 0$

Mass Moment of Inertia →

$I = \frac{mr^2}{2}$

$I = mr^2$

$I = \frac{1}{12}m(a^2 + b^2)$

$I_0 = \frac{1}{3}mL^2$

$I_G = \frac{1}{12}mL^2$

$I = \frac{2}{5}mr^2$

★ When missing equations check $\hat{i}$ & $\hat{j}$ components. ★

$\vec{V}_\theta = \omega\times r$, $a_t = \alpha\times r$, $a_n = \omega\times(\omega\times r)$ ★ SIGNS!!!!!

$a_A = a_B + (a_{A/B})_t + (a_{A/B})_n = a_B + (\alpha\times r_{A/B})_t + (\omega\times\omega\times r_{A/B})_n$

For a Rolling Wheel →

① $\Sigma F_x = ma_{Gx}$ → $P - F$

② $\Sigma F_y = ma_{Gy}$ → $N - mg$

③ $\Sigma M_A = I_G\alpha$ → $Fr = I_G\alpha$

④ Slip → $F = \mu_k mg$

NO Slip → $a_G = \alpha r$

check $F_F < F_S$

Wheel Slipping to Non-Slip →

① $G_1 + \int_{t_1}^{t_2}F\,dt = G_2$

② $H_{0_1} + \int_{t_1}^{t_2}M_0\,dt = H_{0_2}$

Conservation of Linear & Angular Momentum

③ $V_G = \omega\times r$, $a_G = \alpha\times r$

Pulley Equation → $\dot{y}_A + \dot{y}_B + \dot{y}_C + \dots = 0$

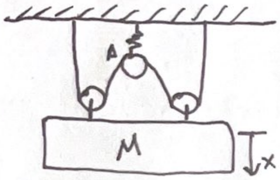
SINE LAW → $\frac{A}{\sin A} + \frac{B}{\sin B} + \frac{C}{\sin C}$

Efficiency → P_{out}/P_{in}

COSINE LAW → $c^2 = a^2 + b^2 - 2ab\cos\theta$

$$R = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{\left|d^2y/dx^2\right|}$$

$x_1 = 2x_2 \Rightarrow N_1 x_1 = N_2 x_2$???



Deflection of mass by x moves A by $2x$

$\therefore T = kx$