ECE110: Electrical Fundamentals (Lecture Notes)

Arnav Patil

University of Toronto

Contents

1 Electricity

1.1 Lecture 01

What is this course about?

- Fundamental physics of electrical engineering
- Basic electronic circuits
- Electronics are everywhere
- Textbook is a custom textbook on Wiley, made for this course

Electric Charges

- Come in positive and negative – opposites attract
- Static charges – stationary electric charges
- Crushing wintergreen lifesavers
- Charges more through friction – excess charges

1.2 Lecture 02

ELectric charges recap

- Negative charges transferred from rod to silk
- Negative charges transferred from fur to rod
- **We only deal with ideal conductors and insulators in this course**

Conductors & Insulators

- Both conductors and insulators are able to carry excess charge

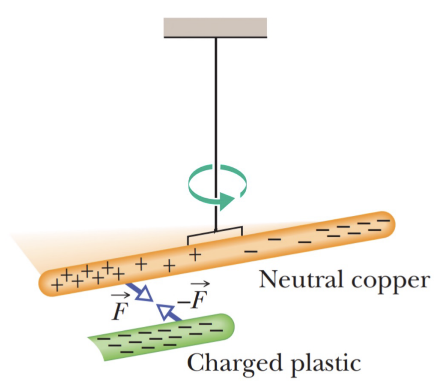

Figure 1: Induced Charge

Coulomb's Law

- Force between two electrically charged particles is called the electrostatic force
  - Quantified by Charles-Augustine de Coulomg in 1785
- SI unit of charge is the coulomb (C)
- We do not consider shape when discussing particles

$$\vec{F} = \frac{kq_1q_2}{r^2}\hat{r} \tag{1}$$

Electrostatic Constant

- $k = \frac{1}{4\pi\varepsilon_0} = 8.99 \times 10^9 Nm^2/C^2$
- ε_0 is the permittivity constant of free space: $8.85 \times 10^{-12} C^2/Nm^2$
- F_{AB} means force exerted on A by B
- Force will be radially towards or exactly away from other particle

Multiple Forces

- If multiple forces are exerted on one particle, F_{NET} is the vector sum of all forces on the said particle
- If $\sum F = 0$, then we have equilibrium

Shell Theories – proved later by Gauss' Law

- Theory 1: charged particle outside a shell with charge uniformly distributed on its surface is attracted/repelled as if shell's charge is concentrated at the centre of the shell
  - Assumption: presence of particle has negligible effect on distribution of charge of shell
- Theory 2: charged particle inside a shell with a charge distributed evenly on its surface has no net force due to the shell
  - Assumption: presence of particle has negligible effect on distribution of charge of shell

Charge is Quantized

- Electric charges come in set quantities – proved by the Millikan Oil Drop Experiment
- Any $q = ne$, where $e = 1.602 \times 10^{-19}$
- Net electric charge of any isolated system is always conserved

1.3 Lecture 03

Electric Fields

- Region in which an electric charge experiences an electric force
  - Coined by Micheal Faraday to describe a force at a distance (without physical contact)
- Electric fields always compared to a **small, positive test charge**.

$$\vec{E} = \frac{\vec{F}}{q_0} \tag{2}$$

Electric Field Lines

- ALWAYS drawn positive to negative
- When there's two particles, the field line drawn at any point is the tangent of the net field at that point

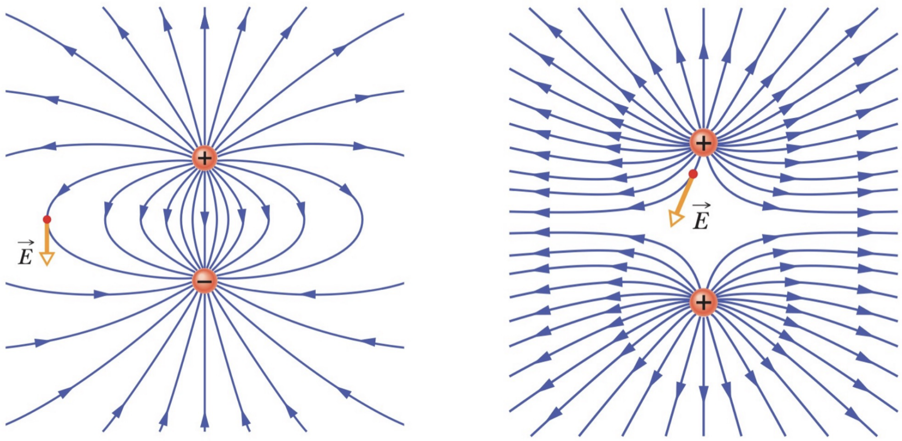

Figure 2: Electric Field Lines

Electric Field Due to a Charged Particle

- Also called a point charge

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \tag{3}$$

- We can take the vector sum of a group of fields to find a net electric field
- If particle with charge q placed in a field $\vec{E}$, then force experienced is $q\vec{E}$

Millikan's Oil Drop Experiment

- Allowed Millikan to find charge to mass ratio
- Charged oil droplets were placed into a charged field and the voltage was varied until the droplets were floating w/ no net acceleration
- Data analysis showed that charge is quantized
  - Jumped by 1.902×10^{-19} gave rise to the elementary charge

1.4 Lecture 04

- Gauss' Law – surface integral of electric field, AKA electric flux of $\vec{E}$ over a closed surface s
  - Equals total charge (w/ sign) enclosed in s, divided by ε_0
  - Assumes that charges can reside in a vacuum.

Flux of Electric Field

- Orientation of surface to field matters
- $\phi = \int E \cdot dA$
- One way to account for surface orientation is to define area of a vector
  - Area vector is vector drawn orthogonal to the surface
- How to evaluate a double integral

$$\int_{y-c}^{y=d} \int_{x=a}^{x=b} f(x)g(y) \cdot dx \cdot dy = \int_{y-c}^{y=d} g(y) \cdot dy \cdot \int_{x=a}^{x=b} f(x) \cdot dx = G(y)\Big|_c^d \cdot F(x)\Big|_a^b \tag{4}$$

Charge Densities

- In many cases the number of charges is so large, we should consider it as a continuous, not discrete distribution of charge
  - Line charge density $\lambda \to C/m$
  - Surface charge density $\sigma \to C/m^2$
  - Volume charge density $\delta \to C/m^3$

1.5 Lecture 05

Electric Flux

- Amount of $\vec{E}$ piercing a small square patch w/ area ΔA defined to be $\Delta\phi$

$$\phi = \sum E \cdot \Delta A \tag{5}$$

- Special case of Gauss' Law: consider a particle w/ charge $+q$ is surrounded by an imaginary concentric sphere

$$\phi = \oint E \cdot dA = E \oint dA = \frac{q}{\varepsilon_0} \tag{6}$$

- Gauss' Law relates the net flux of an electric field through a closed surface and net charge enclosed by the surface:

$$\varepsilon_0 \phi = q_{enc} \tag{7}$$

- Gaussian surface must be a closed surface
- A charge outside of a Gaussian surface contributed zero net flux through the system
- Thus, Gauss' Law can also be written in terms of the electric field piercing the enclosing Gaussian surface

$$\varepsilon_0 \oint E \cdot dA = q_{enc} \tag{8}$$

1.6 Lecture 06

Electric Potential Energy

- Defined as $U = -W_\infty$, where W_∞ is the work that would be done by the electric force on bringing the object from an infinite distance
- Electric potential defined as:

$$V = \frac{W_\infty}{q_0} = \frac{U}{q_0} \tag{9}$$

- If a particle with charge q is placed at a point where the electric potential of a charged object is V, the electric potential energy U is: $U = qV$
- Particle moving from i to f, the electric potential charge is $\Delta V = V_f - V_i$
- Conservation of energy applies to any such closed system

Equipotential Surfaces

- Adjacent points that have the same electric potential form an equipotential surface, which can be either an imaginary surface or a real physical force
  - Like a contour surface map
- A family of EP surfaces associated w/ the electric field dye to some distribution of charges
  - There cannot be a component of $\vec{E}$ that is along the equipotential surface
- In a uniform field:
$$\Delta V = -E\Delta x \tag{10}$$

1.7 Lecture 07

- Positive q – positive PD and vice versa

PD Due to a Group of Particles

- $$V = \sum_{i=1}^{n} V_i = k\sum_{i=1}^{n} \frac{q_i}{r_i} \tag{11}$$
- For 2 particles: $U = k\frac{q_1 q_2}{r}$
- Total E_P for a system of particles is the sum of the potential energies for every pair of particles in the system

Electric Potential Energy of a Two Particle System

- EPE system is equal to work required to assemble system w/ particle initially at rest and infinitely distant from each other
$$U = qV \tag{12}$$
- Total PE of system is sum of potential energies for EVERY PAIR of particles in the system

1.8 Lecture 08

Capacitors

- Two conductors, electrically isolated from each other, and from surroundings, forms a capacitor
  - Surface of conductor is an equipotential system in static conditions
  - The electric field $\vec{E}$ just outside surface of a conductor is perpendicular to surface

Capacitance

- When a capacitor is charged, charges of conductors have same magnitude but opposite charge
- If PD between two plates is V, charge $q \propto V$ – this proportionality is called capacitance
$$q = CV \tag{13}$$
- SI unit of capacitance is called farad (F)

Parallel Plate Capacitor

- PPC consists of two parallel conducting plates w/ area A separated by distance d
- $\vec{E}$ is uniform in central region b/w the plates, but near the edges we observe "fringing."

Charging a Capacitor

- Device that maintains a certain PD between terminals
- We assume capacitors can retain a charge indefinitely
- Until given opportunity to discharge

Calculating Capacitance

1. Assume a q on plates
2. Calculate $\vec{E}$ between plates:
$$\varepsilon_0 \oint E \cdot dA = q \tag{14}$$
3. Calculate PD:
$$V_f - V_i = -\int_i^f E \cdot ds \tag{15}$$
4. Calculate C using $q = CV$

1.9 Lecture 09

Capacitance of a Parallel Plate Capacitor

- Area A and spacing of d – $C = \frac{\varepsilon_0 A}{d}$
- Capacitance of cylindrical capacitor – $C = \frac{2\pi\varepsilon_0 L}{\ln b/a}$
- Capacitance of a spherical capacitor – $C = 4\pi\varepsilon_0 \frac{ab}{b-a}$
- Capacitance of an isolated sphere – $C = 4\pi\varepsilon_0 R$

Capacitors in Parallel

$q_1 = C_1 V$, $q_2 = C_2 V$, $q_3 = C_3 V$ – Total charge $\rightarrow q_{tot} = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$

$$C_{eq} = \frac{q_{tot}}{V} = \frac{(C_1 + C_2 + C_3)V}{V} = C_1 + C_2 + C_3 \tag{16}$$

$$C_{eq} = \sum_{j=1}^{n} C_j \tag{17}$$

Capacitors in Series

$$V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3} \tag{18}$$

$$V = V_1 + V_2 + V_3 = q(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}) \tag{19}$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \tag{20}$$

Potential Energy of a Charged Capacitor

$$U = \frac{1}{2}CV^2 \tag{21}$$

Energy Density

- Energy density of a capacitor is the potential energy stored per unit volume

$$u = \frac{1}{2}\varepsilon_0 E^2 \tag{22}$$

1.10 Lecture 10

Capacitor with a Dielectric

- Must be insulating material with a constant κ
  - $\varepsilon = \kappa\varepsilon_0$
  - $\kappa > 1$ for any dielectric material

Electric Current

- Defined as $\frac{dq}{dt}$
- $i = \int dq = \int_0^t dq$
- Conventional current is a positive charge movement
- $i_0 = i_1 + i_2$

Current Density

- Magnitude of J is equal to current per unit of the cross section of a conductor
- Direction of J is same as velocity of the moving charge if they are positive
- If current is uniform across surface and parallel to dA then

$$i = \int D \cdot dA = J \int dA = JA \tag{23}$$

Drift Spread of Charge Carriers

- When a conductor has a current through it, the conduction e^- move with a drift speed v_d

$$\vec{J} = (ne)\vec{v}_d \tag{24}$$

1.11 Lecture 11

Resistance

- Resistance R is given by $R = \frac{V}{i}$, unit is ohm Ω

Resistivity

- Resistivity ρ of a material we define as $\rho = \frac{E}{J}$
  - Unit is ohm meter $\Omega \cdot m$
  - Vector equation is $\vec{E} = \rho\vec{J}$

Resistance of a Conducting Wire

- Resistance R of conducting wire of length L and uniform cross section A is:

$$R = \rho \frac{L}{A} \tag{25}$$

Conductivity

- Conductivity σ of a material is defined as $\sigma = \frac{1}{\rho}$
- $\vec{J} = \sigma \vec{E}$

Change of ρ with Temperature

- Relation between ρ and temp T is appointed by:

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0) \tag{26}$$

Ohm's Law

- Assertion that current through a device is always directly proportional to PD applied to the device

$$V = iR \tag{27}$$

Power in Electric Current

- Power P is rate of energy transfer
- $P = iV$
- For a resistor w/ resistance R the electrical energy dissipation due to a resistance is:

$$P = i^2 R = \frac{V^2}{R} \tag{28}$$

2 Magnetism

2.1 Lecture 12

Magnetic Field $\vec{B}$

- Magnetic fields can produce magnetic force
- Who produces a magnetic field?
  - Magnetic charges – monopoles theorized to exist but not proven
  - Permanent magnet – intrinsic magnetic field around an object
  - Moving charged particles
- SI unit for $\vec{B}$ is tesla (T) – Gauss (G)
- Array of DOTS represents a magnetic field coming OUT of plane
- Array of ARROWS represents a magnetic field going INTO the plane

Magnetic Force

- When charged particles move in a field, a magnetic field is given by

$$\vec{F} = q(\vec{v} \times \vec{B}) \tag{29}$$

Magnetic Field Lines

- Always go from North to South
- Direction of tangent line gives directions of field vector at that point
- Spacing of lines represents the magnitude of $\vec{B}$

Magnetic Dipole

- Field lines emerge from North pole and enter into South pole
- Magnets always come in dipoles – never just North or South

2.2 Lecture 13

Magnetic Force on a Current Carrying Wire

- Wire exposed to a magnetic field will experience a force

$$\vec{F} = i\vec{L} \times \vec{B} \tag{30}$$

- If the wire is not straight:

$$d\vec{F}_b = i\vec{L} \times \vec{B} \tag{31}$$

Magnetic Field Due to a Current

- Moving charged particles produce a magnetic field

$$dB = \frac{\mu_0}{4\pi}\frac{i \cdot ds \sin\theta}{r^2} \tag{32}$$

- Biot-Savart Law

Magnetic Field due to a Long Straight Wire

- DIRECTION – use right hand rule for current in a wire
- Same points can be applied to analyze a wire of any shape and length

Field Due to Current in a Circular Arc

$$B = \frac{\mu_0 i\phi}{4\pi R} \tag{33}$$

2.3 Lecture 14

Force Between Two Parallel Currents

- Parallel currents attract and opposite currents repel

$$F = \frac{\mu_0 L i_a i_b}{2\pi d} \tag{34}$$

- Rail Guns!!!
- Ampere's Law of field around a wire

$$B = \frac{\mu_0 i}{2\pi R} \tag{35}$$

Ampere's Law

$$\oint \vec{B} \cdot ds = \mu_0 i_{enc} \tag{36}$$

- This line integral represents the total amount current enclosed within an Amperian Loop

Ampere's Law Inside of Long Straight Wire

$$B = \frac{\mu_0 ir}{2\pi R^2} \tag{37}$$

Solenoids and Currents

$$B = \mu_0 in \tag{38}$$

2.4 Lecture 15

Ampere's Law

$$\oint \vec{B} ds = \mu_0 i_{enc} \tag{39}$$

- Solenoid – magnetic version of capacitor – $B = \mu_0 in$

Faraday's Law

First Experiment

- Conducting loop connected to ammeter – should read zero
- If we move a bar magnet toward to loop, it will read some current
  - If stopped will read zero again
  - Moved backwards, and will read negative current
- EMF – electromotive force, **not a real force**

$$\varepsilon = \frac{dW}{dq} \tag{40}$$

- Difference between emf and electric potential – emf is PD not produced by an electric charge
- Discoveries:
  - Current only when relative motion
  - Faster motion – more current

Second Experiment

- Second loop added attached to circuit
- When switch closed, a brief current will be produced in a second loop
  - If kept closed, no current is registered
  - If opened again, then brief current in opposite direction

Key Takeaway

- An emf and a current can be induced in a loop by changing amount of magnetic field passing through the loop
  - MAGNETIC FLUX!!!
- Magnetic Fluc – $\phi_B = \int \vec{B} \cdot dA$, unit is weber (Wb)
- Edge case – loop lies in plane and $\vec{B}$ is perpendicular to plane

$$\phi_B = \vec{B}\vec{A} \tag{41}$$

Faraday's Law

- Faraday's Law of Induction:

$$\varepsilon = -\frac{d\phi_B}{dt} \tag{42}$$

- If we change the loop to a coil of N turns:

$$\varepsilon = -N\frac{d\phi_B}{dt} \tag{43}$$

Lenz's Law

- An induced current will have a direction such that $\vec{B}$ produced by current opposed change in ϕ_B

2.5 Lecture 16

Induction

- Rectangular loop of wire L has one end in a uniform external magnetic field directed perpendicularly

$$\vec{F}_B = i\vec{L} \times \vec{B} \tag{44}$$

- To find current, we apply Faraday's Law

$$\phi_B = BA \tag{45}$$

$$F = \frac{B^2L^2v}{R^2} \tag{46}$$

Work by Induction

- $W = Fd$
- Power:

$$P = \frac{B^2L^2v^2}{R} \tag{47}$$

Eddy Current Loop

- Same phenomenon occurs when we have a solid conducting plane

2.6 Lecture 17

Inductors

- A device that is used to produce a known magnetic field in a specified region
- We typically use a solenoid

Inductance

- $L = \frac{N\phi_B}{i}$ – unit is henry (H)
  - $N\phi_B$ – magnetic field linkage
  - Inductance is a measure of the flux linkage produced by inductor per unit current

Inductance of a Solenoid

- Consider a solenoid of length l and area A
  - $B = \mu_0 in$
  - $L = \mu_0 n^2 lA$
  - $L \propto l, L \propto A, L \propto n^2$

Self-Inductance

- An induced ε_L appears in any coil in which the current is changing

$$\varepsilon_L = -\frac{d(N\phi_B)}{dt} \tag{48}$$

- Self induced ε has an orientation such that it opposes the change in current i.

$$V_L = L\frac{di}{dt} \tag{49}$$

Energy Stored in Magnetic Field

- If an inductor L carries current i, magnetic field is given by:

$$U_B = \frac{1}{2}Li^2 \tag{50}$$

Inductors in Series

$$L_{eq} = \sum_{j=1}^{n} L_j \tag{51}$$

Inductors in Parallel

$$\frac{1}{L_{eq}} = \sum_{j=1}^{n} \frac{1}{L_j} \tag{52}$$

2.7 Lecture 18

Magnetism review for midterm! I'll skip it as it's all already covered.

3 Basics of Circuit Analysis

3.1 Lecture 19

$$i(t) = \frac{dq(t)}{dt} \ \rightarrow \ q(t) = \int_{-\infty}^{t} i(x) \cdot dx \tag{53}$$

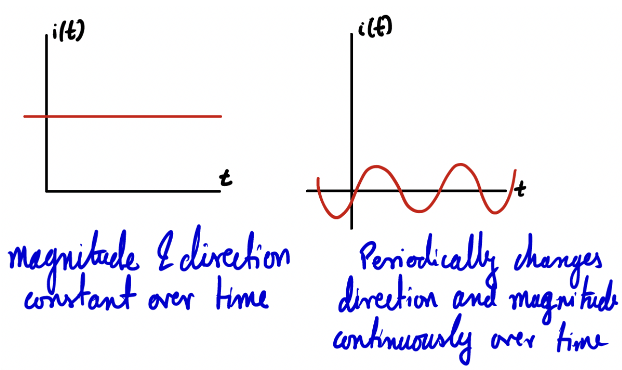

Figure 3: DC and AC Current

Passive Sign Convention

- If product of current and voltage represents magnitude and sign of power dissipated
  - Positive is power absorbed, and
  - Negative is power supplied

Independent Voltage Source

- An independent voltage source is a 2-terminal element that maintains a specified voltage regardless of the current going through it

Independent Current Source

- Sources of current that maintains same current regardless of voltage applied

Dependent Sources

- V/C determined by another point in the circuit
- Four types:
  - voltage controlled voltage source: μ,
  - current controlled voltage source: r,
  - voltage controlled current source: g, and
  - current controlled current source: β.

Tellegen's Theorem

- By the conservation of energy, the power supplied by an energy network equals the power absorbed by the network.

$$P_1 + P_2 + P_3 + ... + P_n = 0 \textbf{ OR } \sum P_i = 0 \tag{54}$$

Ohm's Law and Resistance

- Independent sources do NOT obey Ohm's Law
- For any Ohmic resistor:

$$V = iR \tag{55}$$

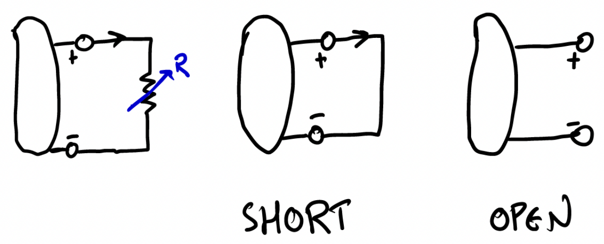

Figure 4: Open and Short Circuits

Conductance

$$G = \frac{1}{R} \tag{56}$$

$$i = GV \tag{57}$$

Power Using Ohm's Law

- General:

$$P = iV \tag{58}$$

- For Ohmic resistors:

$$P = Ri^2 = \frac{V^2}{R} \tag{59}$$

$$P = \frac{i^2}{G} = GV^2 \tag{60}$$

Node, Loop, and Branches

- A node is simply a point in connection of 2/+ circuit elements
- A loop is any closed path through the circuit in which no node is encountered more than once
- A branch is a portion of a circuit containing only a simple element and nodes at each end of the element

3.2 Lecture 20

Kirchoff's Current Law (KCL)

- Algebraic sum of all current entering a node is zero

$$\sum_{j=1}^{N} i_j(t) = 0 \tag{61}$$

Kirchoff's Voltage Law (KVL)

- The algebraic sum of all the voltages around any loop is zero

$$\sum_{j=1}^{N} V_j(t) = 0 \tag{62}$$

Voltage Division Rule

$$V_{P_1} = \frac{R_1}{R_1 + R_2} V(t) \textbf{ and } V_{P_2} = \frac{R_2}{R_1 + R_2} V(t) \tag{63}$$

Current Division Rule

$$i_{P_1} = \frac{R_2}{R_1 + R_2} i(t) \textbf{ and } i_{P_2} = \frac{R_1}{R_1 + R_2} i(t) \tag{64}$$

3.3 Lecture 21

Reference Node or Ground

- Node voltages are often defined with respect to a common point in the circuit
  - This is commonly called the ground
- Usually the node with the largest number of branches connected to it

$$i = \frac{V_m - V_N}{R} \tag{65}$$

Nodal Analysis 1/4 – Circuits Containing Only Independent Current Sources

- If we have N nodes, we need to write $N - 1$ KCL equations
- Write a KCL equation for every non-reference node
- Apply Ohm's Law

$$\begin{bmatrix} 1 & 1-2 & 1-3 \\ 1-2 & 2 & 2-3 \\ 1-3 & 2-3 & 3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \textbf{Current terms (depend on equations)}$$

where the diagonal is the sum of the conductances connected to each node, and non-diagonal values are negatives of conductance between nodes.

Nodal Analysis 2/4 – Circuits Containing Dependent Current Sources

- Same as above, except now we have an extra constraint equation that describes one current in terms of another.

Nodal Analysis 3/4 – Circuits Containing Independent Voltage Sources

- If source is connected to reference node, then node voltage is already given
- If source is connected to two non-reference nodes, then need to use a supernode
  - Write a KCL equation for the full supernode, and
  - Write a constraint equation for the supernode itself

3.4 Lecture 22

Nodal Analysis 4/4 – Circuits With Dependent Voltage Sources

- Same as the above, except with additional constraint equations for the voltage source

3.5 Lecture 23

Loop Analysis

- Just as nodal analysis utilizes KCL equations and node voltages, loop analysis utilizes KVL equations and loop currents
- A mesh is a current loop that doesn't contain another loop within it
- There are $B - N + 1$ linearly independent equations for any network

Loop Analysis 1/3 – Circuits With Only Independent Voltage Sources

- Apply KVL to each mesh
- Apply Ohm's Law
- To determine the sign of the voltage source:
  - Asume positive and if it aids the assumed direction of the current's flow.

Loop Analysis 2/3 – Circuits With Independent Current Sources

- Same thing as above except now we are given the loop current from the beginning

3.6 Lecture 24

Loop Analysis 3/3 – Circuits Containing Dependent Sources

- Same thing as above, except may need supermesh equations, and will have additional constraint equations

How to Know Which Type of Analysis to Use?

- Look at question and givens carefully, may also depend on what kind of value is ultimately needed as the final answer
- Count needed equations:
  - Nodal – $N - 1$
  - Loop – $B - N + 1$

Equivalence

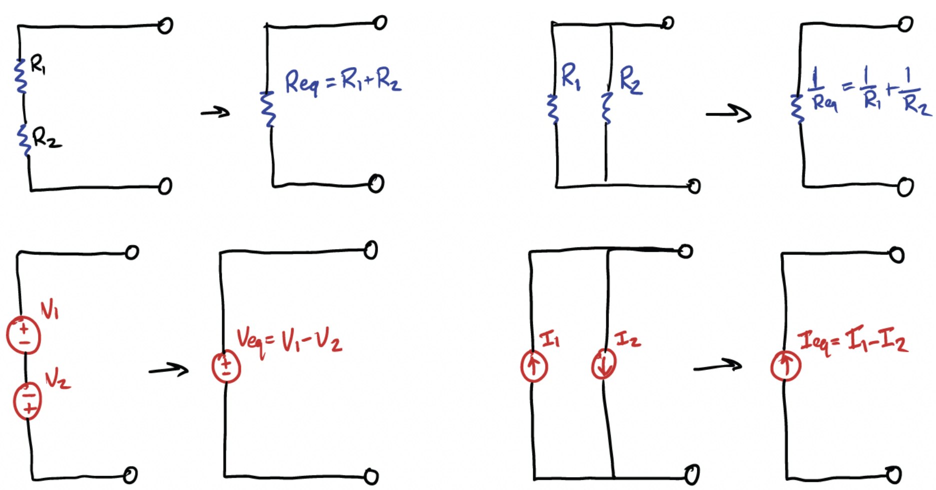

Figure 5: Equivalent Circuits

Linearity

- A linear system satisfies additivity and homogeneity

Superposition

- Current or voltage at any point in the network may be calculated as the algebraic sum of the individual contributions of each source acting alone

3.7 Lecture 25

Lecture 25 was just examples covering previously discussed topics.

3.8 Lecture 26

Thevenin's and Norton's Theorem

- Thevenin's – $V_0 = V_{OC} - R_{Th}i$
- Norton's – $i = -\frac{V_0}{R_{Th}} + i_{SC}$

Circuits Containing Only Independent Sources

- Find the open circuit voltage V_{OC}
- Find the Thevenin resistance R_{Th}

3.9 Lecture 27

Circuits Containing Only Dependent Sources

- An independent voltage or current source is applied at the open terminals and the corresponding current/voltage is measured.
- Voltage/current ratio at the terminal is Thevenin equivalent resistance
- Remember $V_{OC} = 0$ as there is no energy source

3.10 Lecture 28

Circuits Containing Both Independent and Dependent Sources

- Find V_{OC}
- Find i_{SC}
- Calculate R_{Th}

Maximum Power Transfer

Occurs when $R_L = R$:

$$P_L = \frac{V^2}{4R_L} \tag{66}$$

4 Transient Response of Circuits

4.1 Lecture 29

Also has review of Basics of Circuit Analysis.

- In DC analysis, a capacitor creates an open circuit so we don't consider it
- Inductors are short circuits in DC analysis

4.2 Lecture 30

Capacitor/Inductor Review

- Capacitor – open circuit

$$V = V_{t_0} + \frac{1}{C}\int_{t_0}^{t} i(t) \cdot dt \tag{67}$$

- Inductor – short circuit

$$i = i_{t_0} + \frac{1}{L}\int_{t_0}^{t} V(t) \cdot dt \tag{68}$$

- Voltage across capacitor cannot change instantaneously
- Current in inductor cannot change instantaneously

First Order Transient Circuit

- First order – single energy storage element – either capacitor or inductor

First-Order Differential Equation

$$\frac{dx(t)}{dt} + ax(t) = A \tag{69}$$

has solution

$$x(t) = x_c(t) + x_p(t) \tag{70}$$

where $x_p(t)$ is the forced response and $x_c(t)$ is the natural response.

- We assume $x_p(t) = K_1$
- $x_c(t) = K_2e^{-at}$ – $\tau = \frac{1}{a}$ called the time constant
- Therefore we can generalize solution as:

$$x(t) = K_1 + K_2e^{-t/\tau} \tag{71}$$

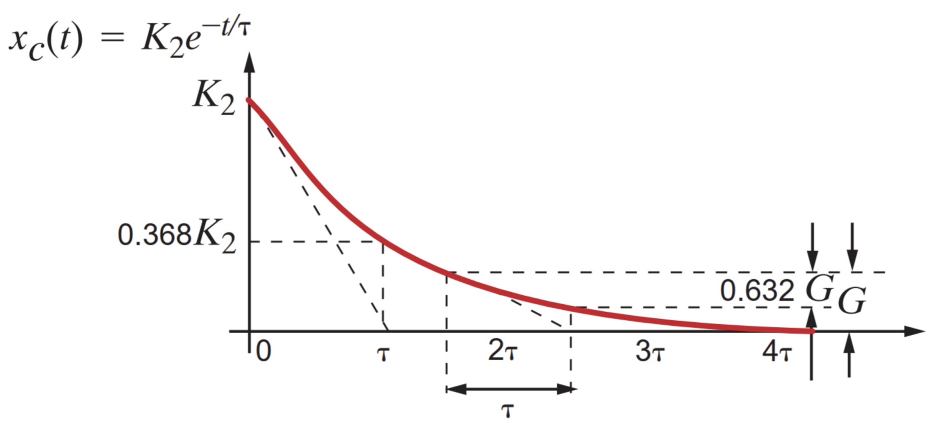

Figure 6: Transient Response

- Larger τ means it takes longer for the circuit to settle

4.3 Lecture 31

5 Steps to solve a circuit equation

1. Solution is always $x(t) = K_1 + K_2 e^{-t/\tau}$
2. Solve for voltage across capacitor, $V_C(0^-)$ or current through inductor $i_L(0^-)$ before the switch is thrown
3. Replace capacitor w/ voltage source $x(0^-) = V_C(0^+)$
4. Solve for steady state 2
$$x(t)\Big|_{t>5\tau} \doteq x(\infty) \tag{72}$$
5. The time constant for:
   - Capacitor:
$$\tau = R_{Th} \cdot C \tag{73}$$
   - Inductor:
$$\tau = \frac{L}{R_{Th}} \tag{74}$$
6. Combine and evaluate constants:
$$x(0^+) = K_1 + K_2 \tag{75}$$
$$x(\infty) = K_1 \tag{76}$$

4.4 Lecture 32

Lecture 32 was just examples covering previously discussed topics.

5 AC Circuit Analysis

5.1 Lecture 33

Sinusoids

- $x(t) = X_M \sin(\omega t)$
- $\omega = \frac{2\pi}{T} = 2\pi f$
- There is also the phase angle – $x(t) = X_M \sin(\omega t + \theta)$
- Positive phase angle moves signal earlier
- For $x_1(t) = X_M \sin(\omega t + \theta)$ and $x_2(t) = X_M \sin(\omega t + \phi)$
  - $x_1(t)$ leads by $(\theta - \phi)$ rads
  - $x_2(t)$ lags by $(\phi - \theta)$ rads
- If $x(t) = X_M \sin(\omega t + \frac{\pi}{2}) = \cos(\omega t)$

Complex Numbers

- Polar – $A = z\angle\theta$
- Rectangular – $A = x + jy$
- Exponential – $A = ze^{j\theta}$
- Euler's Identity – $e^{j\theta} = \cos\theta + j\sin\theta$

5.2 Lecture 34

- If we apply a sinusoidal forcing function to a linear network
  - State voltages and currents in the network will also be sinusoidal

$$V(t) = A\sin(\omega t + \theta) \longrightarrow i(t) = B\sin(\omega t + \phi) \tag{77}$$

- Every steady state voltage/current in a linear network will have same form and same frequency.

Phasors

- Phase angles are based on cosine functions

$$\mathbb{V} = V_M \angle\theta \tag{78}$$

- This complex representation is called a phasor

Phasor Relationship for RLC Elements

- For resistors, V_M and I_M are always in place
- For inductors, voltage always leads current by 90
- For capacitors, current leads voltage by 90

5.3 Lecture 35

Impedance

- Essentially a complex version of resistance – with same unit Ohm
  - Now we are dealing with phasor voltage and current
  - Impedance is not a phasor
- $Z = \frac{\mathbb{V}}{\mathbb{I}}$

$$Z = \frac{V_M \angle\theta_V}{I_M \angle\theta_i} \tag{79}$$

- Expressed in rectangular form as $\mathbb{Z}(\omega) = R(\omega) + jX(\omega)$
- Impedance of:

$$R \rightarrow \mathbb{Z} = R \tag{80}$$

$$C \rightarrow \mathbb{Z} = \frac{1}{j\omega C} \tag{81}$$

$$L \rightarrow \mathbb{Z} = j\omega L \tag{82}$$

5.4 Lecture 36

Lecture 36 was just examples covering previously discussed topics.

5.5 Lecture 37

Lecture 37 was just examples covering previously discussed topics.

5.6 Lecture 38

Lecture 38 was a final exam review lecture, no new content was discussed.